

Anisotropic fluid for a set of non-diagonal tetrads in $f(T)$ gravity

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Abstract

We consider $f(T)$ gravity for a Weitzenböck spherically symmetric and static spacetime, where the metric is projected in the tangent space to the manifold for a set of non-diagonal tetrads. The matter content is coupled through the energy momentum tensor of an anisotropic fluid, generating various classes of new black hole and wormhole solutions. One of these classes is that of cold black holes. We also perform the reconstruction scheme of the algebraic function $f(T)$ for two cases where the radial pressure is proportional to $f(T)$ and its first derivative.

Pacs numbers: 04.50. Kd, 04.70.Bw, 04.20. Jb

1 Introduction

A special attention is now attached to the so-called Teleparallel Theory (TT). This is a geometric theory which possesses only torsion, without curvature, for characterising the gravitational interaction between matter fields. The TT is equivalent to the General Relativity (GR) [1], and the Weitzenböck connection [2] which generates the torsion of the spacetime, whose the contraction defines the action of the theory, which is invariant under local Lorentz transformation [3].

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With the progress of the measurements about the evolution of the universe, as the expansion and the acceleration, the dark matter and dark energy, various proposals for modifying the GR are being tested. As the unifications theories, for the scales of low energies, it appears in the effective actions besides the Ricci scalar, the terms R^2 , $R^{\mu\nu}R_{\mu\nu}$ and $R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$ and the proposal of modified gravity that agrees with the cosmological and astrophysical data is $f(R)$ theory [4, 5]. The main problem that one faces with this theory is that the equation of motion is of order 4, being more complicated than the GR for any analyse. Since the GR possesses the TT as analogous, it has been thought the called $f(T)$ gravity, T being the torsion scalar, which would be the analogous to the generalizes GR, the $f(R)$ gravity. The $f(T)$ gravity is the generalization of the TT as we shall see later. Note also that the $f(T)$ theory is free of the curvature, i.e., it is defined from the Weitzenbock connection. However, it has been shown recently that this theory breaks the invariance of the local Lorentz transformations [6]. Other recent problem is that the $f(T)$ gravity appears to be dependent on the used frame, i.e., it is not covariant [6, 7].

The $f(T)$ gravity appears in Cosmology as the source driving the inflation [8]. Also, it has been used as a theory that reproduces the acceleration of the universe without the necessity of introducing the dark energy [9, 10, 11]. The contributions of the cosmological perturbations of this theory have been studied [12]. In gravitation, the first model of black hole which has been analysed was the BTZ [13]. Recently, also in the framework of $f(T)$ gravity, several black hole and wormhole solutions with spherical symmetry have been found [14, 15, 16]. Moreover, other analysis about various themes are being investigated in the context of $f(T)$ gravity [22].

In this paper, we study some solutions obtained by fixing the spherical symmetries and the staticity of the metric, as well as the more usual methods of GR [17]. Introducing an anisotropic matter content, we obtain new black hole and traversable wormholes solutions for the $f(T)$ gravity, considering a set of non-diagonal tetrads.

The paper is organized as follows. In Section 2, we will present a brief revision of the fundamental concepts of the Weitzenbok geometry, the action of the $f(T)$ gravity and the equations of motion. In Section 3, we will fix the symmetries of the geometry and present the equations of the energy density, the radial and tangential pressures. The Section 4 is devoted for obtaining new solutions in the $f(T)$ gravity. In Section 5, we present a summary of the reconstruction method for the static case of the theory $f(T)$ and reconstruct two simplest cases linked with the radial pressure. The conclusion and perspectives are presented in the Section 6.

2 The field equations from $f(T)$ theory

The mathematical concept of the $f(T)$ gravity is based on the Weitzenbock's geometry and there exist some excellent works in this way [3, 18, 19]. Our convention and nomenclature are the following:

the Latin index describe the elements of the tangent space to the manifold (spacetime), while the Greek ones the elements of the spacetime. For a general spacetimes metric, we can define the line element as

$$dS^2 = g_{\mu\nu} dx^\mu dx^\nu . \quad (1)$$

This metric can be projected in the tangent space to the manifold, using the representation of the tetrad matrix, where the line element is

$$dS^2 = g_{\mu\nu} dx^\mu dx^\nu = \eta_{ij} \theta^i \theta^j , \quad (2)$$

$$dx^\mu = e_i{}^\mu \theta^i , \quad \theta^i = e^i{}_\mu dx^\mu , \quad (3)$$

where $\eta_{ij} = \text{diag}[1, -1, -1, -1]$ and $e_i{}^\mu e^i{}_\nu = \delta_\nu^\mu$ or $e_i{}^\mu e^j{}_\mu = \delta_i^j$. The square root of the metric determinant is given by $\sqrt{-g} = \det[e^i{}_\mu] = e$. Now, we describe the spacetime through the tetrad matrix, and then define the Weitzenbock connection as

$$\Gamma_{\mu\nu}^\alpha = e_i{}^\alpha \partial_\nu e^i{}_\mu = -e^i{}_\mu \partial_\nu e_i{}^\alpha . \quad (4)$$

From the definition of connection (4), the spacetime possesses a curvature identically null, having only the torsion contribution and its related quantities for this geometry. Due to the fact that the antisymmetric part of the connection does not vanish, we can define directly from the components of the connection, the torsion tensor, whose components are given by

$$T_{\mu\nu}^\alpha = \Gamma_{\nu\mu}^\alpha - \Gamma_{\mu\nu}^\alpha = e_i{}^\alpha (\partial_\mu e^i{}_\nu - \partial_\nu e^i{}_\mu) . \quad (5)$$

Through the torsion tensor, we can define two important tensor in this geometry, the contorsion and the tensor S whose components can be written as

$$K^{\mu\nu}{}_\alpha = -\frac{1}{2} (T^{\mu\nu}{}_\alpha - T^{\nu\mu}{}_\alpha - T_\alpha{}^{\mu\nu}) , \quad (6)$$

$$S_\alpha{}^{\mu\nu} = \frac{1}{2} (K^{\mu\nu}{}_\alpha + \delta_\alpha^\mu T^{\beta\nu}{}_\beta - \delta_\alpha^\nu T^{\beta\mu}{}_\beta) . \quad (7)$$

We are now able to define easily the scalar that makes up the action of $f(T)$ gravity, the torsion scalar T . Through (5)-(7), we define the scalar torsion scalar as

$$T = T_{\mu\nu}^\alpha S_\alpha{}^{\mu\nu} . \quad (8)$$

Following the same idea of the GR and $f(R)$ theory for the coupling of the geometry with the matter part, we define the action of the $f(T)$ gravity as

$$S[e_\mu^i, \Phi_A] = \int d^4x e \left[\frac{1}{16\pi} f(T) + \mathcal{L}_{\text{Matter}}(\Phi_A) \right] , \quad (9)$$

where we used the units in which $G = c = 1$ and the Φ_A are the matter fields. Considering the action (9) as a functional of the fields e_μ^i and Φ_A , and vanishing the variation of the functional with respect to

the field e^i_ν , which is the principle of minimum action, one obtains the following equation of motion [6]

$$S_\mu^{\nu\rho}\partial_\rho T f_{TT} + [e^{-1}e_\mu^i\partial_\rho(ee_i^\alpha S_\alpha^{\nu\rho}) + T^\alpha_{\lambda\mu}S_\alpha^{\nu\lambda}] f_T + \frac{1}{4}\delta_\mu^\nu f = 4\pi\mathcal{T}_\mu^\nu, \quad (10)$$

where \mathcal{T}_μ^ν is the energy momentum tensor, $f_T = df(T)/dT$ and $f_{TT} = d^2f(T)/dT^2$. If we consider $f(T) = a_1T + a_0$, the TT is recovered with a cosmological constant.

We now introduce the matter content as being described by an anisotropic fluid, whose energy-momentum components are given by

$$\mathcal{T}_\mu^\nu = (\rho + p_t) u_\mu u^\nu - p_t \delta_\mu^\nu + (p_r - p_t) v_\mu v^\nu, \quad (11)$$

where u^μ is the four-velocity, v^μ the unit space-like vector in the radial direction, ρ the energy density, p_r the pressure in the direction of v^μ (radial pressure) and p_t the pressure orthogonal to v_μ (tangential pressure). Since we are assuming an anisotropic spherically symmetric matter, one has $p_r \neq p_t$, such that their equality corresponds to an isotropic fluid sphere.

In the next section, we will make some considerations for the manifold symmetries in order to obtain simplifications in the equation of motion and the specific solutions of these symmetries.

3 Spherically Symmetric geometry

The line element of a spherically symmetric and static spacetimes can be described, without loss of generality, as

$$dS^2 = e^{a(r)} dt^2 - e^{b(r)} dr^2 - r^2 (d\theta^2 + \sin^2(\theta) d\phi^2). \quad (12)$$

In order to re-write the line element (12) into the invariant form under the Lorentz transformations as in (2), we define a set of non-diagonal tetrads (3) as

$$\{e^i_\mu\} = \begin{pmatrix} e^{a/2} & 0 & 0 & 0 \\ 0 & e^{b/2} \sin \theta \cos \phi & r \cos \theta \cos \phi & -r \sin \theta \sin \phi \\ 0 & e^{b/2} \sin \theta \sin \phi & r \cos \theta \sin \phi & r \sin \theta \cos \phi \\ 0 & e^{b/2} \cos \theta & -r \sin \theta & 0 \end{pmatrix}. \quad (13)$$

This choice of tetrad matrices is not unique, because the aim of letting the line element invariant under local Lorentz transformations, resulting in the form (2). Other choices have been performed with diagonal matrices, as in references [15, 16]. Using (13), one can obtain $e = \det[e^i_\mu] = e^{(a+b)/2} r^2 \sin(\theta)$, and with (4)-(8), we determine the torsion scalar in terms of r

$$T(r) = \frac{2e^{-b}}{r^2} (e^{b/2} - 1) (e^{b/2} - 1 - ra'), \quad (14)$$

where the prime (') denote the derivative with respect to the radial coordinate r . One can now re-write the equations of motion (10) for an anisotropic fluid as

$$4\pi\rho = \frac{f}{4} - \left(\frac{T}{4} - \frac{1}{r^2} - \frac{e^{-b}}{r^2} (1 - rb') \right) \frac{f_T}{2} - \frac{e^{-b/2}}{r} (e^{-b/2} - 1) (f_T)' , \quad (15)$$

$$4\pi p_r = \left(\frac{T}{2} + \frac{e^{-b}}{r^2} (1 + ra') - \frac{1}{r^2} \right) \frac{f_T}{2} - \frac{f}{4} , \quad (16)$$

$$4\pi p_t = \frac{e^{-b}}{2} \left(\frac{a'}{2} + \frac{1}{r} - \frac{e^{b/2}}{r} \right) (f_T)' + \left[\frac{T}{2} + e^{-b} \left(\frac{a''}{2} + \left(\frac{a'}{4} + \frac{1}{2r} \right) (a' - b') \right) \right] \frac{f_T}{2} - \frac{f}{4} , \quad (17)$$

where p_r and p_t are the radial and tangential pressures respectively. In the case where there does not exist a set of non-diagonal, as in [15, 16], we get an off-diagonal equation (component $\theta - r$) which imposed to the algebraic function $f(T)$ to be a linear function of T , or the torsion scalar had to be a constant, with a free $f(T)$. Now, in this present paper, with the choice of a set of non-diagonal tetrads, we do not get such constraint equation, but rather, $f(T)$ may assume an arbitrary functional form. In the next section, we will determine new solutions for the $f(T)$ theory making some consideration about the matter component $p_r(r)$.

4 New solutions for a set of non-diagonal tetrads

In this section, we will study two simplest cases for a set of non-diagonal tetrads. The first is when the radial pressure p_r , in (16), is identically null. This has been done originally in GR by Florides [17] and used later by Boehmer et al [20], and in $f(T)$ gravity, the same condition has been used for the cases of diagonal tetrads [15, 16]. We can classify these two cases:

1. Considering $f(T) = \sum_n a_n T^n$ and $p_r = 0$ in (16), we get

$$\sum_n T^n \left[a_n - (n+1)a_{n+1} \left(T - \frac{2}{r^2} + \frac{2}{r^2} e^{-b} (1 + ra') \right) \right] = 0 , \quad (18)$$

which yields

$$T(r) = \frac{a_n}{(n+1)a_{n+1}} + \frac{2}{r^2} [1 - e^{-b} (1 + ra')] . \quad (19)$$

We have three principal cases here:

- (a) When we make the choice of the quasi-global coordinate

$$a(r) = -b(r) , \quad (20)$$

equating (14) with (19), one gets

$$\begin{aligned} e^{-a(r)} &= e^{b(r)} = 12 \left\{ 6a_{n+1}^2 (n+1)^2 r^2 + a_{n+1} (n+1) r (a_n r^3 + 3c_0) + \right. \\ &\quad \left. + 2\sqrt{3} \sqrt{a_{n+1}^3 (n+1)^3 r^3 [3a_{n+1} (n+1) r + a_n r^3 + 3c_0]} \right\} \\ &\quad / (a_n r^3 + 3c_0)^2 . \end{aligned} \quad (21)$$

This is a black hole solution, whose horizon is $r_H = \sqrt[3]{-3c_0/a_n}$, with $\text{sign}[c_0/a_n] = -1$.

(b) When we impose

$$e^{a(r)} = \left(1 - \frac{r_H}{r}\right)^m, \quad (22)$$

with $m \geq 2$, equating (14) with (19), one obtains

$$e^{b(r)} = \frac{2}{a_n^2 r^4 (r - r_H)^2} \left\{ a_{n+1}^2 (n+1)^2 [2r + (m-2)r_H]^2 + 2a_n a_{n+1} (n+1) r^2 (r - r_H) [r + (m-1)r_H] - \sqrt{x(r)} \right\}, \quad (23)$$

$$x(r) = a_{n+1}^3 (n+1)^3 [2r + (m-2)r_H]^2 \times \\ \times \left(a_{n+1} (n+1) [2r + (m-2)r_H]^2 + 4a_n r^2 (r - r_H) [r + (m-1)r_H] \right). \quad (24)$$

This is the first class of cold black holes solutions in $f(T)$ gravity. The degenerated event horizon of order m is obtained in $r = r_H$.

(c) With the choice

$$e^{b(r)} = \frac{r_0}{r}, \quad (25)$$

and equating (14) with (19), one gets

$$a(r) = c_0 - \frac{1}{16(n+1)a_{n+1}} \left\{ -4r_0 a_n r \left(1 + \sqrt{\frac{r_0}{r}} \right) + 32(n+1)a_{n+1} \ln(r) + 2(16(n+1)a_{n+1} + a_n r_0^2) \left(\arctan \left(2\sqrt{\frac{r}{r_0}} \right) - \ln(4r - r_0) \right) \right\}. \quad (26)$$

This solution is completely different of that obtained in [16], taking the same condition of coordinate but for a set of diagonal tetrads.

2. When the radial pressure is simply a function of the radial coordinate r and the algebraic function $f(T)$ is chosen as

$$f(T) = a_2 T^2 + a_1 T + a_0. \quad (27)$$

In this case, substituting (27) into (16), and taking the coordinate condition [16]

$$a(r) = \ln \left(\frac{r_0}{r} \right), \quad (28)$$

we obtain

$$T(r) = \frac{2}{r^2} \left[1 \pm \sqrt{1 - \frac{a_1}{2a_2} r^2 + \frac{r^4}{4a_2} (a_0 + 16\pi p_r(r))} \right]. \quad (29)$$

Equating (29) with (14), taking into account (28), we get the solution

$$dS^2 = \frac{r_0}{r} dt^2 - \left[1 - \frac{r^2}{2} T(r) \right]^{-2} dr^2 - r^2 d\Omega^2, \quad (30)$$

where $T(r)$ is given in (29). This is a new class of traversable wormhole solutions. We can observe this by following the same process as in [15]. This line element (12) can be put in the form

$$dS^2 = e^{a(r)} dt^2 - dl^2 - r^2(l) d\Omega^2, \quad (31)$$

where $a(r)$ is denoted redshift function, and through the redefinition $\beta(r) = r[1 - e^{-b(r)}]$, with $b(r)$ being the metric function given in (12), $\beta(r)$ is called shape function. Therefore, the conditions of existence of a traversable wormhole are: a) the function $r(l)$ must possess a minimum value r_1 for r , which imposes $d^2r(l)/dl^2 > 0$; b) $\beta(r_1) = r_1$; c) $a(r_1)$ has a finite value; and finally d) $d\beta(r)/dr|_{r=r_1} \leq 1$.

In this case, $\beta(r) = r[1 - (1 - r^2T(r)/2)^2]$. Hence, the conditions of a traversable wormhole applied on the torsion scalar $T(r)$, in (29). The condition a), which can be explained in $d^2r/dl^2 = [\beta(r) - r\beta'(r)]/2r^2 = -r[1 - r^2T(r)/2][T(r) + rT'(r)/2] > 0$, leads to $[T(r) + rT'(r)/2] < 0$; the condition b), yields $T(r_1) = 2/r_1^2$; the condition c) is always satisfied, as we can see through (30); and finally, the condition d) leads to $[1 - r^2T(r)/2] \geq 2r^2[T(r) + rT'(r)/2]$, which is always true when a) is satisfied.

Therefore, we must choose the torsion scalar, which will be defined from the radial pressure $p_r(r)$, in (29), such that it satisfies the conditions a)-d), for obtaining traversable wormhole solutions. Let us consider first a class of solutions coming from the following example, $T(r) = 2r_1^{m-2}/r^m$, with $r_1 > 0$ e $m \geq 3$, which is obtained for $p_r(r) = [2a_1r^{2m}r_1^4 - a_0r^{2(m+1)}r_1^4 - 8a_2r^mr_1^{m+2} + 4a_2r^2r_1^{2m}]/16\pi r_1^4r^{2(m+1)}$. The condition a) is satisfied, since $d^2r/dl^2 = (r_1^{m-2}/r^{m-1})[1 - (r_1/r)^{m-2}] > 0$; the conditions b) and c) are also directly satisfied; and d) is satisfied through a).

5 Reconstruction in static f(T) theory

A method widely used in cosmology is called reconstruction. This method stems from the introduction of an auxiliary field for the reconstruction of the algebraic function of the main action, as in the case of the theory $f(R)$ for example [21].

We can briefly present this method as follows. Considering the algebraic function

$$f(T) = P(\varphi)T + Q(\varphi), \quad (32)$$

the functional variation of the action (9), with respect to φ , is given by

$$\frac{\delta S}{\delta \varphi} = \frac{e}{16\pi} \left[\frac{dP}{d\varphi}T + \frac{dQ}{d\varphi} \right] = 0. \quad (33)$$

Solving this equation, we get $\varphi \equiv \varphi(T)$, then, $f(T) = P[\varphi(T)]T + Q[\varphi(T)]$. Hence, we have the following

identities

$$f_T(T) = P + \left(\frac{dP}{d\varphi} T + \frac{dQ}{d\varphi} \right) \frac{d\varphi}{dT} = P[\varphi(T)] , \quad (34)$$

$$f_{TT}(T) = \frac{dP[\varphi(T)]}{dT} . \quad (35)$$

Having in hand the equations (32), (34) and (35), and substituting into (15)-(17), we get

$$4\pi\rho = -\frac{e^{-b/2}}{r} \left(e^{-b/2} - 1 \right) \frac{dP}{dT} + \frac{Q}{4} + \frac{P}{2r^2} [1 + e^{-b}(1 - rb')] , \quad (36)$$

$$4\pi p_r = \frac{P}{2r^2} [-1 + e^{-b}(1 + ra')] - \frac{Q}{4} , \quad (37)$$

$$4\pi p_t = \frac{e^{-b}}{2} \left(\frac{a'}{2} + \frac{1}{r} - \frac{e^{-b/2}}{r} \right) \frac{dP}{dT} + P \frac{e^{-b}}{4} \left[a'' + \left(\frac{1}{r} + \frac{a'}{2} \right) (a' - b') \right] - \frac{Q}{4} . \quad (38)$$

The reconstruction can be performed directly, making use of the auxiliary field $\varphi = r$. This method may be used for re-obtaining or reconstructing, when the inversion $r \equiv r(T)$ is possible, for the $f(T)$ theory in the static case.

Let us draw up to cases here:

1. For the case where $f(T)$ is a linear algebraic function, the have the condition $P \in \mathfrak{R}$, due to (35).

Considering now the condition (28), and that the radial pressure (37), obeys

$$p_r = g_1 f(T) , \quad (39)$$

where $g_1 \in \mathfrak{R}$, we get

$$Q = -\left(\frac{2}{16\pi g_1 + 1} \right) \frac{P}{r^2} - \left(\frac{16\pi g_1}{16\pi g_1 + 1} \right) PT \quad (40)$$

$$f(T) = \left(\frac{1}{16\pi g_1 + 1} \right) PT - \left(\frac{2}{16\pi g_1 + 1} \right) \frac{P}{r^2} . \quad (41)$$

Differentiating (41) with respect to T , equating to P and integrating, we obtain the relation

$$r(T) = [8\pi g_1 (T_0 - T)]^{-1/2} , \quad (42)$$

where T_0 is an integration real constant. Substituting (42) in (41), the algebraic function $f(T)$ is then constructed as

$$f(T) = PT + \left(\frac{-16\pi g_1 P T_0}{16\pi g_1 + 1} \right) . \quad (43)$$

We see that Q is also a constant, given by the second term of (43). Substituting (14) into (42), one gets the following solution

$$dS^2 = \frac{r_0}{r} dt^2 - \left(\frac{16\pi g_1 + 1}{16\pi g_1} - \frac{T_0}{2} r^2 \right)^{-2} dr^2 - r^2 d\Omega^2 . \quad (44)$$

This is a new traversable wormhole solution. Following the same process of the solution (30), we obtain $\beta(r) = r[1 - (k - T_0 r^2/2)^2]$, with $k = (16\pi g_1 + 1)/16\pi g_1$, and then, the minimum of r is given by $r_1 = \sqrt{2k/T_0}$. The condition a), which in this case is given by $d^2 r/dt^2 = -T_0 r(k - T_0 r^2/2)$, and satisfied for $T_0 < 0$ and $-1/16\pi < g_1 < 0$. The conditions b) and c) are directly satisfied, and d) is also satisfied for $\beta'(r_1) = 1$.

2. Taking the condition (28), and considering that the radial pressure obeys [16]

$$4\pi p_r = g_1 f(T) + g_2 f_T(T) , \quad (45)$$

where $g_1, g_2 \in \mathfrak{R}$, we get from (37) that

$$Q = -\frac{4P}{4g_1 + 1} \left(g_1 T + g_2 + \frac{1}{r^2} \right) , \quad f(T) = \frac{4P}{4g_1 + 1} \left(\frac{T}{4} - g_2 - \frac{1}{2r^2} \right) . \quad (46)$$

Differentiating (46) with respect to T and equating it to P , we get

$$g_1 P = \frac{dP}{dT} \left(\frac{T}{4} - g_2 - \frac{1}{2r^2} \right) + \frac{P}{r^3} \frac{dr}{dT} . \quad (47)$$

In order to integrate this equation, we consider first $P(r) = r^k$, then, (47) becomes

$$g_1 r = \left[\frac{k}{4}(T - 4g_2) + \frac{1}{r^2} \left(1 - \frac{k}{2} \right) \right] \frac{dr}{dT} , \quad (48)$$

which is separable only for $k = 2$, yielding

$$r(T) = r_1 \left(\frac{T - 4g_2}{T_0 - 4g_2} \right)^{2g_1} , \quad (49)$$

where T_0 and r_1 are integration real constants. Substituting (49) into (46), we have reconstructed the algebraic function $f(T)$, as

$$f(T) = \frac{1}{4g_1 + 1} \left[r_1^2 \frac{(T - 4g_2)^{4g_1 + 1}}{(T_0 - 4g_2)^{4g_1}} - 2 \right] . \quad (50)$$

This algebraic polynomial function $f(T)$, can be better extended, when a particular more simplest case. One can illustrate the particular case in which $4g_1 + 1 = 2$, we get

$$f(T) = a_1 T + (a_2 T^2 + a_0) , \quad (51)$$

where $a_2 = r_1^2/2(T_0 - 4g_2)$, $a_1 = -4g_2 r_1^2/(T_0 - 4g_2)$ and $a_0 = -1$. Then, we see that our reconstruction scheme leads to a function which presents the contributions of higher orders for the torsion scalar, added to the TT, which in this case is of second and zero orders in (51).

6 Conclusions

The particular choice of the representation of a spherically symmetric and static metric of a spacetime of Weitzenböck can be done in an arbitrary way, when we wish to maintain the invariance under local

Lorentz transformations. Therefore, when we choose a set of diagonal tetrads, for a projection in the tangent space to the manifold, the imposition of the function $f(T)$ being linear in T , or a constant torsion scalar, is recovered. But, as described in this work, choosing a set of non-diagonal tetrads, we can get an arbitrary algebraic function $f(T)$. Therewith, the second derivative of the function $f(T)$, appears in the contribution of the energy density and the tangential pressure in (15) and (17).

We shown some examples of new black holes and wormholes solutions. Through the fixation of the null radial pressure and the coordinates conditions (20)-(25), we illustrated an example of a black hole, (21), of a cold black hole, in (23), and a solution for comparing the same conditions, with the diagonal case, in (26).

We also imposed to the radial pressure to be just function of the radial coordinate, and $f(T)$ to be a polynomial function of second order in T . We then obtained, through the coordinate condition (28), a new class of traversable wormholes solution in (30).

Finally, we made a summary of the reconstruction scheme for our static case of $f(T)$ gravity. For a particular case of the algebraic function $f(T)$ being a linear function, we were able to reconstruct it, considering that it is proportional the radial pressure. This case is that of a new solution of traversable wormhole. Taking into account the condition in which the radial pressure is a linear combination of the algebraic function $f(T)$ and its derivative, we reconstructed $f(T)$ as polynomial function of the torsion scalar T , where we shown the particular case in which $f(T)$ gravity would be the TT plus some terms of second and zero orders for the torsion scalar T .

Acknowledgement: M. H. Daouda thanks CNPq/TWAS for financial support. M. E. Rodrigues thanks UFES for the hospitality during the development of this work.

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